1 standard deviation is 12 points ($\sigma = 12$). Therefore, X is half of 12 points from the mean, or

$$(0.5)(12) = 6 \text{ points}$$

STEP 3 Find the X value.

Starting with the value of the mean, use the direction (step 1) and the distance (step 2) to determine the X value. For this demonstration, we want to find the score that is 6 points below $\mu = 60$. Therefore,

$$X = 60 - 6 = 54$$

Formula 5.2 is used to convert a z-score to an X value. For this demonstration, we obtain the following, using the formula:

$$\begin{aligned} X &= \mu + z\sigma \\ &= 60 + (-0.50)(12) \\ &= 60 + (-6) = 60 - 6 \\ &= 54 \end{aligned}$$

Notice that the sign of the z-score determines whether the deviation score is added to or subtracted from the mean.

PROBLEMS

1. Describe exactly what information is provided by a z-score.

2. A distribution has a standard deviation of $\sigma = 4$. Find the z-score for each of the following locations in the distribution.
   a. Above the mean by 4 points
   b. Above the mean by 12 points
   c. Below the mean by 2 points
   d. Below the mean by 8 points

3. A distribution has a standard deviation of $\sigma = 10$. For each of the following z-scores, determine whether the location is above or below the mean and determine how many points away from the mean. For example, $z = +1.00$ corresponds to a location that is above the mean by 10 points.
   a. $z = +2.00$
   b. $z = +0.50$
   c. $z = -2.00$
   d. $z = -1.50$

4. For a population with $\mu = 80$ and $\sigma = 20$,
   a. Find the z-score for each of the following X values. (*Note:* You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

   $X = 75 \quad X = 90 \quad X = 110$

   $X = 95 \quad X = 60 \quad X = 40$

   b. Find the score (X value) that corresponds to each of the following z-scores. (*Note:* You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

   $z = 2.50 \quad z = -0.50 \quad z = -1.50$

   $z = 0.25 \quad z = -0.75 \quad z = 1.00$

5. For a population with $\mu = 45$ and $\sigma = 7$, find the z-score for each of the following X values. (*Note:* You probably will need to use the formula and a calculator to find these values.)

   $X = 47 \quad X = 35 \quad X = 40$

   $X = 60 \quad X = 55 \quad X = 42$

6. A population has a mean of $\mu = 50$ and a standard deviation of $\sigma = 10$.
   a. For this population, find the z-score corresponding to each of the following scores.

   $X = 55$ $X = 40$ $X = 35$

   $X = 48$ $X = 70$ $X = 65$

   b. For the same population, find the score (X value) corresponding to each of the following z-scores.

   $z = -2.00$ $z = 1.50$ $z = -0.50$

   $z = 0.60$ $z = 1.00$ $z = 0$

7. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 8$.
   a. For this population, find the z-score corresponding to each of the following scores.

   $X = 74$ $X = 68$ $X = 86$

   $X = 62$ $X = 82$ $X = 54$

   b. For the same population, find the score (X value) corresponding to each of the following z-scores.

   $z = 0.75$ $z = 1.50$ $z = 2.50$

   $z = -1.00$ $z = -0.25$ $z = -3.00$

8. A sample of $n = 25$ scores has a mean of $M = 60$ and a standard deviation of $s = 12$. Find the z-score corresponding to each of the following scores from this sample.

   $X = 66$ $X = 48$ $X = 84$

   $X = 55$ $X = 70$ $X = 75$

9. A sample has a mean of $M = 75$ and a standard deviation of $s = 10$. Find the X value corresponding to each of the following z-scores for this sample.

   $z = 1.50$ $z = -2.30$ $z = -0.80$

   $z = 0.40$ $z = -1.20$ $z = 2.10$

10. Find the z-score corresponding to a score of $X = 50$ for each of the following distributions.
    a. $\mu = 60$ and $\sigma = 5$
    b. $\mu = 40$ and $\sigma = 5$
    c. $\mu = 60$ and $\sigma = 20$
    d. $\mu = 40$ and $\sigma = 20$

11. Find the X value corresponding to $z = +1.50$ for each of the following distributions.
    a. $\mu = 100$ and $\sigma = 10$
    b. $\mu = 100$ and $\sigma = 20$
    c. $\mu = 80$ and $\sigma = 4$
    d. $\mu = 80$ and $\sigma = 2$

12. A score that is 10 points below the mean corresponds to a z-score of $z = -2.00$. What is the population standard deviation?

13. A score that is 8 points above the mean corresponds to a z-score of $z = 0.50$. What is the population standard deviation?

14. For a population with a standard deviation of $\sigma = 4$, a score of $X = 44$ corresponds to $z = -0.50$. What is the population mean?

15. For a sample with a standard deviation of $s = 10$, a score of $X = 65$ corresponds to $z = 1.50$. What is the sample mean?

16. For a sample with a mean of $M = 85$, a score of $X = 90$ corresponds to a z-score of $z = 1.00$. What is the sample standard deviation?

17. For a population with a mean of $\mu = 70$, a score of 62 corresponds to a z-score of $z = -2.00$. What is the population standard deviation?

18. In a population of exam scores, a score of $X = 88$ corresponds to $z = +2.00$ and a score of $X = 79$ corresponds to $z = -1.00$. Find the mean and standard deviation for the population. (*Hint:* Sketch the distribution and locate the two scores on your sketch.)

19. In a distribution of scores, $X = 62$ corresponds to $z = +0.50$, and $X = 52$ corresponds to $z = -2.00$. Find the mean and standard deviation for the distribution.

20. For each of the following populations, would a score of $X = 48$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?
    a. $\mu = 40$ and $\sigma = 10$ b. $\mu = 40$ and $\sigma = 2$
    c. $\mu = 50$ and $\sigma = 4$ d. $\mu = 60$ and $\sigma = 4$

21. Suppose that you have a score of $X = 55$ on an exam with $\mu = 50$. Which standard deviation would give you the better grade: $\sigma = 5$ or $\sigma = 10$?

22. Answer the question in Problem 21, but this time assume that the mean for the exam is $\mu = 60$.

23. On Tuesday afternoon, Bill earned a score of $X = 73$ on an English test with $\mu = 65$ and $\sigma = 8$. The same day, John earned a score of $X = 63$ on a math test with $\mu = 57$ and $\sigma = 3$. Who should expect the better grade, Bill or John? Explain your answer.

24. Suppose that you got a score of $X = 78$ on an English test for which the mean was $\mu = 70$ and the standard deviation was $\sigma = 10$. Also, suppose that you got a score of $X = 64$ on a Spanish test with $\mu = 50$ and $\sigma = 7$. For which test would you expect the better grade? Explain your answer.

25. A distribution with a mean of $\mu = 38$ and a standard deviation of $\sigma = 4$ is being transformed into a standardized distribution with $\mu = 50$ and $\sigma = 10$. Find the new, standardized score for each of the following values from the original population.

a. $X = 42$ **b.** $X = 40$
c. $X = 38$ **d.** $X = 36$

26. A distribution with a mean of $\mu = 86$ and a standard deviation of $\sigma = 12$ is being transformed into a standardized distribution with $\mu = 100$ and $\sigma = 20$. Find the new, standardized score for each of the following values from the original population.

a. $X = 80$ **b.** $X = 89$
c. $X = 95$ **d.** $X = 98$

27. A population consists of the following $N = 6$ scores: 0, 4, 6, 1, 3, and 4.

a. Compute μ and σ for the population.
b. Find the z-score for each score in the population.
c. Transform the original population into a new population of $N = 6$ scores with a mean of $\mu = 50$ and a standard deviation of $\sigma = 10$.

28. A population consists of the following $N = 5$ scores: 0, 6, 4, 3, and 12.

a. Compute μ and σ for the population.
b. Find the z-score for each score in the population.
c. Transform the original population into a new population of $N = 5$ scores with a mean of $\mu = 60$ and a standard deviation of $\sigma = 8$.